Course

Electrons, phonons and photons :

momentum :

$$p = \frac{h}{\lambda}$$

or

$$p = \hbar K$$

- photons : λ is very large (400 - 700 nm) ⇒ momentum is negligible
- phonons : λ is very small (few angstoms) ⇒ large momentum

wavevector :

$$K = \frac{2\pi}{\lambda}$$

- photons. $K \sim 0$
- phonons : k is large

Energy :

Photons : $E \sim 1.7 - 3.1$ eV in the visible range ⇒ large energy

phonons : $E \leq 63$ meV in silicon ⇒ small energy

Generation and Recombination

Definitions

$G_{0,i}$ — equilib. (0), each process (i)

G_i : generation rate $[cm^{-3}/s]$

R_i : recombination rate $[cm^{-3}/s]$

G_0 — total

G, R : total.

optical :

generation : generation of carriers (not light)

$$G_{0,rad} = g_{rad}(T) \quad [cm^{-3}/s]$$: only depends on T

recombination :

$$R_{0,rad} = \underbrace{r_{rad}(T)}_{} \cdot n_0 \cdot p_0$$

constant depending on material and temperature (how easy it is to emit a photon)

$\boxed{\text{very small in Si} \quad r_{rad} = 2 \times 10^{-15}\ cm^3/s}$

in GaAs at 300K : $r_{rad} = 7 \times 10^{-10}\ cm^3/s$

In TE :

$$G_{0,rad} = R_{0,rad} \Rightarrow g_{rad}(T) = \underbrace{r_{rad}(T)\, n_0 p_0}_{} = r_{rad}(T) \cdot n_i^2$$

For example :

Consider n-type Si with $N_D = 10^{17}\ cm^{-3}$ at R.T. What are the generation and recombination rates due to band-to-band optical processes?

In TE : $G_{0,rad} = R_{0,rad}$

for $N_D = 10^{17}\ cm^{-3} \Rightarrow n_0 \simeq 10^{17}\ cm^{-3}$ thus $p_0 \simeq 10^3\ cm^{-3}$

$n_0 \cdot p_0 = n_i^2$

$n_i \approx 10^{10}\ cm^{-3}$

$$\therefore G_{0,rad} = R_{0,rad} = r_{rad} \cdot n_0 \cdot p_0 =$$

$$= 2 \times 10^{-15} \frac{cm^3}{s} \cdot 10^{17}\ cm^{-3} \cdot 10^3\ cm^{-3} =$$

$$= \underline{2 \times 10^5\ cm^{-3}/s}$$

Out of equilibrium situation:

Optical generation rate: still the same

$$G_{rad} = g_{rad} = r_{rad} \cdot n_0 \cdot p_0$$

Optical recombination rate:

$$R_{rad} = r_{rad} \cdot \underbrace{n \cdot p}_{\text{not in equilibrium}}$$

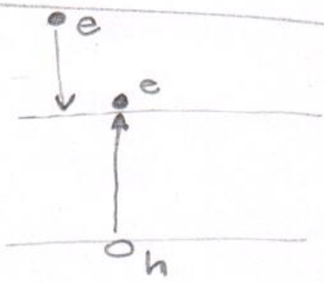
Net recombination rate:

$$U_{rad} = R_{rad} - G_{rad} = r_{rad}\,(np - \underbrace{n_0 p_0}_{n_i^2})$$

$$\begin{cases} \text{if: } n \cdot p > n_i^2 \Rightarrow \text{recombination} \\ n \cdot p < n_i^2 \Rightarrow \text{generation} \end{cases}$$

Auger recombination:

generation:



generation: $G_{0,\,eeh} = g_{eeh}(T) \cdot n_0$

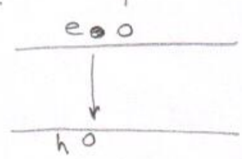
equil. — $G_{0,\,eeh}$; electron - electron - hole — eeh

proportional to number of hot electrons, which is related to number of electrons

recombination: $R_{0,\,eeh} = \underbrace{r_{eeh}}_{\text{constant depending on temperature and material}} \cdot n_0^2 \cdot p_0$

$$\boxed{\text{For Si at R.T.: } r_{eech} = 1.8 \times 10^{-31}\ cm^6/s}$$

recombination:



in TE: $g_{eeh} = r_{eeh} \cdot n_0 p_0$

For example: n-type Si, $N_D = 10^{17}\ cm^{-3}$ at room temperature

r_{eeh} for Si is $1.8 \times 10^{-31}\ cm^6/s$

$$G_{0,\,eeh} = R_{0,\,eeh} = r_{eeh} \cdot n_0^2 p_0 = 1.8 \times 10^{-31} \frac{cm^6}{s} \cdot 10^{17}\ cm^{-3} \cdot 10^{3}\ cm^{-3} =$$

$$= 1.8 \times 10^{6}\ cm^{-3}/s$$

out of equilibrium :

$$G_{eeh} = g_{eeh} \cdot n$$
$$R_{eeh} = r_{eeh} \cdot n^2 \cdot p$$
$$\Bigg\} \; U_{eeh} = R_{eeh} - G_{eeh} = r_{eeh} \cdot n \left(np - n_i^2\right)$$

Trap - assisted :

* Derivation is uploaded on moodle.

⊛ exercise to demonstrate this

for $E_t = E_i$ with N_t

Probability of occupation : $f(E_t) = f(E_i) = \dfrac{1}{1 + \exp\left(\frac{E_i - E_F}{kT}\right)} = \dfrac{n_i}{n_i + p_0}$

. occupied traps : $N_t \times f(E_i) = n_{t0}$

. Unoccupied traps : $N_t - \left(N_t\, f(E_i)\right) = \left(N_t - n_{t0}\right)$

Rates :

capture of electrons : $r_{0,ec} = c_e \cdot n_0 \cdot (N_t - n_{t0})$

electron capture coefficient   # electrons   # available traps

electron emission :

$r_{0,ee} = e_e \cdot n_{t0}$

electron emission coefficient   # occupied traps

* conduction band is largely empty of electrons

hole capture :

$r_{0,hc} = c_h \cdot p_0 \cdot n_{t0}$

hole capture   # holes   # occupied traps

hole emission :

$r_{0,he} = e_h \cdot (N_t - n_{t0})$

* valence band has enough electrons.

for n-type semiconductor:

$$n_i \gg p_o$$

$$\Rightarrow \quad r_{o,ec} = r_{o,ee} \approx \frac{n_i}{\tau_{eo}}$$

$$r_{o,hc} = r_{o,he} \simeq \frac{p_o}{\tau_{ho}}$$

if τ_{eo} not too different from τ_{ho} $\Rightarrow$ $\underbrace{r_{o,e\substack{c\\e}} \gg r_{o,h\substack{c\\e}}}$

. traps communicate more with CB than with VB

Exercise : . n-type Sc with $N_D = 10^{17}\,cm^{-3}$ at room temperature

. There is a dominant trap at E_i with $N_t = 10^{16}\,cm^{-3}$

. For this trap ; $c_e = c_h = 10^{-11}\,cm^3/s$

(a) Rate at which trap-assisted generation and recombination takes place?

(b) Occupation probability of trap?

$$\tau_{eo} = \frac{1}{N_t\, c_e} = \frac{1}{10^{16} \times 10^{-11}} = 10^{-5}\,s$$

$$\tau_{ho} = \frac{1}{N_t\, c_h} = \frac{1}{10^{16} \times 10^{-11}} = 10^{-5}\,s$$

. rate of exchange of electrons between traps and CV :

$$r_{o,ce} = r_{o,ee} \simeq \frac{n_i}{\tau_{eo}} = \frac{10^{10}\,cm^{-3}}{10^{-5}\,s} = 10^{15}\,cm^{-3}/s$$

. for holes

$$r_{o,ch} = r_{o,eh} = \frac{p_o}{\tau_{ho}} = \frac{10^3}{10^{-5}} = 10^8\,cm^{-3}/s$$

Even though the trap is at E_i, $c_e = c_h$, trap "communicates" 7-orders of magnitude more with CV. $\Rightarrow$ because it is a <u>n-type material.</u>

b) $f(E_t) = f(E_i) = \frac{n_i}{n_i + p_0} = \frac{10^{10}}{10^{10} + 10^{3}} \simeq 1$

E_F is well above E_i.

Very useful exercise:

Calculate n_i for Si at room temperature:

$$n_i = \sqrt{N_c \cdot N_v} \cdot \exp\left(\frac{-E_g}{2kT}\right)$$

$$= \sqrt{2.86 \times 10^{19}\ cm^{-3} \times 3.10 \times 10^{19}\ cm^{-3}} \cdot \exp\left(-\frac{1.124\ eV}{2 \cdot 25.9 \times 10^{-3}\ eV}\right) = 1.12 \times 10^{10}\ cm^{-3}$$

Useful to keep this value in mind

$$\boxed{n_i = 1.12 \times 10^{10}\ cm^{-3}} \approx 10^{10}\ cm^{-3} \text{ for quick calculations}$$

Proof 2 :

$$e_e = C_e \cdot n_0 \cdot \frac{N_t - n_{t0}}{n_{t0}}$$

but we know :

$$n_i^2 = n_0 \cdot p_0$$

$$n_{t0} = N_t \cdot \frac{n_i}{n_i + p_0}$$

So :

$$e_e = C_e \cdot \frac{n_i^2}{p_0} \cdot \frac{\cancel{N_t}\left(1 - \frac{n_i}{n_i + p_0}\right)}{\cancel{N_t} \cdot \left(\frac{n_i}{n_i + p_0}\right)} =$$

$$= C_e \cdot \frac{n_i^{\cancel{2}}}{\cancel{p_0}} \cdot \frac{\frac{\cancel{p_0}}{\cancel{n_i + p_0}}}{\frac{\cancel{n_i}}{\cancel{n_i + p_0}}} = C_e \cdot n_i //$$

Recombination and generation:

$T.E \Rightarrow p.n = n_i^2$

out of equilibrium $\Rightarrow$ $pn > n_i^2 \Rightarrow$ recombination

$p.n < n_i^2 \Rightarrow$ generation

Recombination rate: $R = r_{rad} \cdot p . n$

$r_{rad} \approx 10^{-10} \, cm^3/s$ direct band gap

$\approx 10^{-15} \, cm^3/s$ indirect band gap (unlikely)

Net transition rate:

$$U = R - G = r_{rad} \, (p.n - n_i^2)$$

Under low injection: $(\Delta p' << N_D)$

n-type semiconductor:

$n \simeq N_D$

$p = p_0 + \Delta p'$

$\Rightarrow U = r_{rad} \, (\underbrace{\cancel{n.p}}_{n_i^2} + n \cdot p' - \cancel{n_i^2})$

$$U \simeq r_{rad} \cdot N_D \cdot p' \underset{Def.}{\equiv} \frac{p'}{\tau_p}$$

extra # carriers per time.

Definition of lifetime:

Average time for a minority carrier to recombine:

$$\begin{cases} \tau_p = \dfrac{1}{r_{rad} \cdot N_D} & : \text{n-type} \\ \tau_n = \dfrac{1}{r_{rad} \cdot N_A} & : \text{p-type} \end{cases}$$

Exercise :

- consider n-type Si , under low level approximation, and $N_D = 10^{17}\ cm^{-3}$
- Assume a trap at E_i with $N_t = 10^{16}\ cm^{-3}$ and $C_e = C_h = 10^{-11}\ cm^3/s$

$\Rightarrow n_0 \sim N_D = 10^{17}\ cm^{-3}$

we know that:

$$\tau_{rad} = \frac{1}{r_{rad} \cdot n_0} = \frac{1}{2\times 10^{-15}\ cm^3/s \times 10^{17}\ cm^{-3}} = 5ms$$

$$\tau_{Auger} = \frac{1}{K_{eeh} \cdot n_0^2} = \frac{1}{1.8\times 10^{-31}\ cm^6/s \times (10^{17}\ cm^{-3})^2} = 0.56\ ms$$

$$\tau_{tr} = \tau_{h0} = \frac{1}{10^{16}\ cm^{-3} \times 10^{-11}\ cm^3/s} = 10\mu s$$

thus

$$\tau = \frac{1}{\frac{1}{\tau_{rad}} + \frac{1}{\tau_{Auger}} + \frac{1}{\tau_{tr}}} =$$

$$= \frac{1}{\frac{1}{5\times 10^{-3}} + \frac{1}{5.6\times 10^{-4}} + \frac{1}{10^{-5}}} = \underline{\underline{9.8\mu s}}$$ $\Rightarrow$ Recombination lifetime is Dominated by trap-assisted recombinations.

Course 6

$$\frac{dn}{dt} = \frac{dp}{dt} = G - R$$

if $G > R \Rightarrow \frac{dn}{dt} > 0 \, , \, \frac{dp}{dt} > 0 \Rightarrow$ we generate carriers n and p

with external generation (for example shining light):

$$G = G_{ext} + G_{int}$$

then:

$$\frac{dn}{dt} = \frac{dp}{dt} = G_{ext} \underbrace{- G_{int} - R}_{U} = G_{ext} - U$$

Under low-level injection:

$U_i \simeq \frac{n'}{\tau_i}$, $\frac{dn}{dt} = \frac{dn'}{dt}$, since $n = n_0 + n'$ and assuming that n_0 is time independent.

$$\boxed{\frac{dn'}{dt} = G_{ext} - \frac{n'}{\tau}}$$

Case 1: Static case where $\frac{dn}{dt} = 0$ (nothing changes as a function of time)

$$\Rightarrow n' = G_{ext} \cdot \tau$$

excess carriers is the generation rate times the carrier lifetime.

Case 2: if $G_{ext} = 0$

the homogeneous solution is $n' \propto e^{-t/\tau}$: excess carriers decay with a time τ

1) Turn-on transient :

- Lights are turned ON at $t = 0$:

$$\begin{cases} t = 0, & n' = 0 \\ t = \infty, & n' = g_e \cdot \tau \end{cases}$$

- τ is a characteristic time of the problem

$$\Rightarrow \boxed{n'(t) = g_e \tau \left(1 - e^{-t/\tau}\right)}$$

- $t \gg \tau$: rate of generation of carriers decrease towards 0

1. Initially : n' increases since $G > R$

2. As n' builds up, R increases but G does not change.

3. n' is constant when $G = R$ $\Rightarrow$ steady-state is reached!

2) turn-off transient

- Lights are ON for $t < 0$

- $t > 0$, $G_{ext} = 0$

boundary conditions:

$$\begin{cases} t = 0, & n' = g_e \tau \\ t = \infty, & n' = 0 \quad \text{(lights are OFF)} \end{cases}$$

$$\Rightarrow \boxed{n'(t) = g_e \tau e^{-t/\tau}}$$

1. After equilibrium is disrupted, it takes $t > \tau$ to re-establish it.

2. After equilibrium is disrupted : $\underline{\underline{R}}$ not balanced by $(G_{ext} + G_{int})$

$\Rightarrow R > G$, thus n' is reducing and so R

until $R = G$